Etudes de fonctions:
(niveau II^{es})

Etudier les fonctions f suivantes:
($domf, domf'$, limites aux bornes de $domf$, $f'(x)$ sous forme factorisée):

1. $f(x) = \dfrac{x^2}{x+2}$
2. $f(x) = x - 1 + \dfrac{9}{x+1}$
3. $f(x) = \dfrac{4x^2+4x+5}{4x+2}$
4. $f(x) = \dfrac{x^2+2x+2}{x-1}$
5. $f(x) = \dfrac{-x^2-2x-1}{x+3}$
6. $f(x) = \dfrac{x^2}{|x|+2}$
7. $f(x) = -2x + 1 + \dfrac{2}{x-1}$
8. $f(x) = |x+1| + \dfrac{1}{x-1}$
9. $f(x) = \dfrac{(x-2)^2}{x^2+2}$
10. $f(x) = \dfrac{x^2-4x+6}{(x-2)^2}$
11. $f(x) = \dfrac{2x^2-3}{x^2-1}$

Solutions:

1. $f(x) = \dfrac{x^2}{x+2}$

   $domf = \mathbb{R}\backslash\{-2\} = domf'$

   $\lim\limits_{x\to-\infty}\left(\dfrac{x^2}{x+2}\right) = -\infty$

   $\lim\limits_{x\to+\infty}\left(\dfrac{x^2}{x+2}\right) = +\infty$

   $\lim\limits_{x\to-2^-}\left(\dfrac{x^2}{x+2}\right) = -\infty$

   $\lim\limits_{x\to-2^+}\left(\dfrac{x^2}{x+2}\right) = +\infty$

   $f'(x) = \left(\dfrac{x^2}{x+2}\right)' = \dfrac{x(x+4)}{(x+2)^2}$

2. $f(x) = x - 1 + \dfrac{9}{x+1}$

   $domf = \mathbb{R}\backslash\{-1\} = domf'$

   $\lim\limits_{x\to-\infty}\left(x-1+\dfrac{9}{x+1}\right) = -\infty$

   $\lim\limits_{x\to+\infty}\left(x-1+\dfrac{9}{x+1}\right) = +\infty$

   $\lim\limits_{x\to-1^-}\left(x-1+\dfrac{9}{x+1}\right) = -\infty$

   $\lim\limits_{x\to-1^+}\left(x-1+\dfrac{9}{x+1}\right) = +\infty$

   $f'(x) = \left(x-1+\dfrac{9}{x+1}\right)' = \dfrac{(x-2)(x+4)}{(x+1)^2}$

3. $f(x) = \dfrac{4x^2+4x+5}{4x+2}$

   $domf = \mathbb{R}\backslash\{-\dfrac{1}{2}\} = domf'$

$$\lim_{x\to-\infty}\left(\frac{4x^2+4x+5}{4x+2}\right)=-\infty$$

$$\lim_{x\to+\infty}\left(\frac{4x^2+4x+5}{4x+2}\right)=+\infty$$

$$\lim_{x\to-\frac{1}{2}^-}\left(\frac{4x^2+4x+5}{4x+2}\right)==-\infty$$

$$\lim_{x\to-\frac{1}{2}^+}\left(\frac{4x^2+4x+5}{4x+2}\right)=+\infty$$

$$f'(x)=\left(\frac{4x^2+4x+5}{4x+2}\right)'=\frac{(2x-1)(2x+3)}{(2x+1)^2}$$

4. $f(x)=\dfrac{x^2+2x+2}{x-1}$

$domf=\mathbb{R}\backslash\{1\}=domf'$

$$\lim_{x\to-\infty}\left(\frac{x^2+2x+2}{x-1}\right)=-\infty$$

$$\lim_{x\to+\infty}\left(\frac{x^2+2x+2}{x-1}\right)=+\infty$$

$$\lim_{x\to1^-}\left(\frac{x^2+2x+2}{x-1}\right)=-\infty$$

$$\lim_{x\to1^+}\left(\frac{x^2+2x+2}{x-1}\right)=+\infty$$

$$f'(x)=\left(\frac{x^2+2x+2}{x-1}\right)'=\frac{x^2-4-2x}{(x-1)^2}$$

5. $f(x)=\dfrac{-x^2-2x-1}{x+3}$

$domf=\mathbb{R}\backslash\{-3\}=domf'$

$$\lim_{x\to-\infty}\left(\frac{-x^2-2x-1}{x+3}\right)=+\infty$$

$$\lim_{x\to+\infty}\left(\frac{-x^2-2x-1}{x+3}\right)=-\infty$$

$$\lim_{x\to-3^-}\left(\frac{-x^2-2x-1}{x+3}\right)=+\infty$$

$$\lim_{x\to -3^+}\left(\frac{-x^2-2x-1}{x+3}\right)=-\infty$$

$$f'(x)=\left(\frac{-x^2-2x-1}{x+3}\right)'=-\frac{(x+1)(x+5)}{(x+3)^2}$$

6. $f(x)=\dfrac{x^2}{|x|+2}$

$domf=\mathbb{R}$

$domf'=\mathbb{R}$

$$\lim_{x\to -\infty}\left(\frac{x^2}{|x|+2}\right)=+\infty=\lim_{x\to +\infty}\left(\frac{x^2}{|x|+2}\right)$$

$$f(x)=\begin{cases}\dfrac{x^2}{-x+2} & \text{si x } \leq 0\\ \dfrac{x^2}{x+2} & \text{si x} \geq 0\end{cases}$$

f est dérivable en 0 et $f'(0)=0$!!

$$f'(x)=\begin{cases}-\dfrac{x(x-4)}{(x-2)^2} & \text{si x } \leq 0\\ \dfrac{x(x+4)}{(x+2)^2} & \text{si x} \geq 0\end{cases}$$

7. $f(x)=-2x+1+\dfrac{2}{x-1}$

$dom\ f=\mathbb{R}\backslash\{1\}=dom\ f'$

$$\lim_{x\to -\infty}\left(-2x+1+\frac{2}{x-1}\right)=+\infty$$

$$\lim_{x\to 1^-}\left(-2x+1+\frac{2}{x-1}\right)=-\infty$$

$$\lim_{x\to 1^+}\left(-2x+1+\frac{2}{x-1}\right)=+\infty$$

$$\lim_{x\to +\infty}\left(-2x+1+\frac{2}{x-1}\right)=-\infty$$

$$f'(x)=\left(-2x+1+\frac{2}{x-1}\right)'=-\frac{2(x^2-2x+2)}{(x-1)^2}$$

8. $f(x)=\left(|x+1|+\dfrac{1}{x-1}\right)$

$dom\ f=\mathbb{R}\backslash\{1\}$

$dom\ f'=\mathbb{R}\backslash\{1,-1\}$

$\lim\limits_{x\to-\infty}\left(|x+1|+\dfrac{1}{x-1}\right)=+\infty=\lim\limits_{x\to+\infty}\left(|x+1|+\dfrac{1}{x-1}\right)$

$\lim\limits_{x\to1^-}\left(|x+1|+\dfrac{1}{x-1}\right)=-\infty$

$\lim\limits_{x\to1^+}\left(|x+1|+\dfrac{1}{x-1}\right)=+\infty$

$$f(x)=\begin{cases}-x-1+\dfrac{1}{x-1} & \text{si } x\leq-1\\ x+1+\dfrac{1}{x-1} & \text{si } x\geq-1 \text{ et } x\neq1\end{cases}$$

$$f'(x)=\begin{cases}-1-\dfrac{1}{(x-1)^2}=-\dfrac{x^2-2x+2}{(x-1)^2} & \text{si } x<-1\\ 1-\dfrac{1}{(x-1)^2}=\dfrac{(x-2)x}{(x-1)^2} & \text{si } x>-1 \text{ et } x\neq1\end{cases}$$

9. $f(x)=\dfrac{(x-2)^2}{x^2+2}$

$dom\ f=\mathbb{R}=dom f'$

$\lim\limits_{x\to-\infty}\left(\dfrac{(x-2)^2}{x^2+2}\right)=1=\lim\limits_{x\to+\infty}\left(\dfrac{(x-2)^2}{x^2+2}\right)$

$f'(x)=\left(\dfrac{(x-2)^2}{x^2+2}\right)'=\dfrac{4(x-2)(x+1)}{(x^2+2)^2}$

10. $f(x)=\dfrac{x^2-4x+6}{(x-2)^2}$

$dom\ f=\mathbb{R}\backslash\{2\}=dom f'$

$\lim\limits_{x\to-\infty}\left(\dfrac{x^2-4x+6}{(x-2)^2}\right)=1=\lim\limits_{x\to+\infty}\left(\dfrac{x^2-4x+6}{(x-2)^2}\right)$

$\lim\limits_{x\to2^-}\left(\dfrac{x^2-4x+6}{(x-2)^2}\right)=+\infty=\lim\limits_{x\to2^+}\left(\dfrac{x^2-4x+6}{(x-2)^2}\right)$

$$f'(x) = \left(\frac{x^2-4x+6}{(x-2)^2}\right)' = -\frac{4}{(x-2)^3}$$

11. $f(x) = \dfrac{2x^2-3}{x^2-1}$

$dom\ f = \mathbb{R}\backslash\{-1;1\} = dom f'$

$$\lim_{x\to-\infty}\left(\frac{2x^2-3}{x^2-1}\right) = 2 = \lim_{x\to+\infty}\left(\frac{2x^2-3}{x^2-1}\right)$$

$$\lim_{x\to-1^-}\left(\frac{2x^2-3}{x^2-1}\right) == -\infty$$

$$\lim_{x\to-1^+}\left(\frac{2x^2-3}{x^2-1}\right) = +\infty$$

$$\lim_{x\to1^-}\left(\frac{2x^2-3}{x^2-1}\right) = +\infty$$

$$\lim_{x\to1^+}\left(\frac{2x^2-3}{x^2-1}\right) = -\infty$$

$$f'(x) = \left(\frac{2x^2-3}{x^2-1}\right)' = \frac{2x}{(x^2-1)^2}$$

Mise en page des énoncés et
rédaction et mise en page des solutions:

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