

Limites

Exercice 1:

1. Étudier les limites suivantes :

1. $\lim_{x \rightarrow -\infty} (x^3 - 2x + 5)$

2. $\lim_{x \rightarrow -\infty} (x^4 + 2x)$

3. $\lim_{x \rightarrow +\infty} (x^3 - x^4)$

4. $\lim_{x \rightarrow -1} (x^3 - 5x + 1)$

5. $\lim_{x \rightarrow +\infty} \left(\frac{-3}{x^2+5}\right)$

6. $\lim_{x \rightarrow 0} \left(\frac{x^2}{x+2}\right)$

7. $\lim_{x \rightarrow 1} \left(\frac{x^2-1}{x^2+3x+5}\right)$

8. $\lim_{x \rightarrow 0} \left(\frac{x^3-1}{x^2}\right)$

9. $\lim_{x \rightarrow 2} \left(\frac{x^2+5}{(x-2)^2}\right)$

10. $\lim_{x \rightarrow 1} \left(\frac{3-x}{x^2-2x+1}\right)$

11. $\lim_{x \rightarrow 1} \left(\frac{3x-2}{x+1}\right)$

12. $\lim_{x \rightarrow +\infty} \left(\frac{2x^2+5}{x^2-3x+1}\right)$

13. $\lim_{x \rightarrow +\infty} \left(\frac{3x^2+5x}{1-3x^2}\right)$

14. $\lim_{x \rightarrow +\infty} \left(\frac{x+3}{2x^2+5}\right)$

15. $\lim_{x \rightarrow -\infty} \left(\frac{3-x}{x^3+4x+1}\right)$

16. $\lim_{x \rightarrow -\infty} \left(\frac{x^4+3x}{x+3}\right)$

17. $\lim_{x \rightarrow +\infty} \left(\frac{x^2+2x+1}{x}\right)$

18. $\lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2+3x+5}\right)$

19. $\lim_{x \rightarrow -\infty} \left(\frac{x^2+1}{1-x} \right)$

20. $\lim_{x \rightarrow 0} \left(\frac{x^3-3x}{x^2+x} \right)$

21. $\lim_{x \rightarrow 1} \left(\frac{x^2-x}{x^2+2x-3} \right)$

22. $\lim_{x \rightarrow -2} \left(\frac{x^2+4x+4}{x^2-x-6} \right)$

23. $\lim_{x \rightarrow 2} \left(\frac{3x+5}{(x-2)^2} \right)$

24. $\lim_{x \rightarrow -3} \left(\frac{1-3x}{(3+x)^2} \right)$

25. $\lim_{x \rightarrow 1} \left(\frac{x-1}{\sqrt{x}-1} \right)$

26. $\lim_{x \rightarrow 4} \left(\frac{2-\sqrt{x}}{4-x} \right)$

Exercice 2:

Déterminer les limites à gauche et à droite , lorsque x tend vers x_0 , des fonctions f dans les cas suivants :

1. $x_o = 2; f(x) = \frac{3x-7}{x-2}$

2. $x_o = -3; f(x) = \frac{5x}{x+3}$

3. $x_o = -2; f(x) = \frac{-3x+2}{x+2}$

4. $x_o = 1; f(x) = \frac{3x}{1-x}$

5. $x_o = 2; f(x) = \frac{2x-5}{2-x}$

6. $x_o = 0; f(x) = \frac{x^2+3x+5}{x^2+2x}$

7. $x_o = -2; f(x) = \frac{x^2+3x+5}{x^2+2x}$

8. $x_o = 1; f(x) = \frac{2x}{(x-1)(x+3)}$

9. $x_o = -3; f(x) = \frac{2x+1}{(x-1)(x+3)}$

$$10. x_o = -1; f(x) = \frac{3x+7}{(x+1)(2-x)}$$

$$11. x_o = 3; f(x) = \frac{x+2}{6+x-x^2}$$

Exercice 3:

Déterminer les limites suivantes :

$$1. \lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{x^2 - 3x + 2}$$

$$2. \lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{2x^3 - 3x^2 + 1}$$

$$3. \lim_{x \rightarrow 2} \left[\frac{x+3}{x^2+3x-10} \left(\frac{8}{5(x+2)} - \frac{4}{x} + \frac{17x+6}{5(x^2+1)} \right) \right]$$

$$4. \lim_{x \rightarrow 1} \frac{3x-2-\sqrt{4x^2-x-2}}{x^2-3x+2}$$

$$5. \lim_{x \rightarrow 0} \frac{\sqrt{x+1}-\sqrt{x^2+x+1}}{x}$$

$$6. \lim_{x \rightarrow 1} \frac{\sqrt{x+s}-\sqrt{3x+1}}{\sqrt{x-1}}$$

$$7. \lim_{x \rightarrow 2} \frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}$$

$$8. \lim_{x \rightarrow 0} \frac{x^4+1-\sqrt{1-x^4}}{x^2\sqrt{1-x^4}}$$

$$9. \lim_{x \rightarrow 1} \frac{\sqrt{3+\sqrt{2x-1}}-2}{\sqrt{2+\sqrt{3x+1}}-\sqrt{x+3}}$$

$$10. \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{1+x^2}-1}$$

$$11. \lim_{x \rightarrow 0^-} \frac{x}{\sqrt{1+x^2}-1}$$

$$12. \lim_{x \rightarrow +\infty} \frac{(3x^2+1)(5x+3)}{(2x^3-1)(x+4)}$$

$$13. \lim_{x \rightarrow +\infty} \frac{(2x-3)^2(4x+7)^3}{(3x-4)^2(5x^2+1)}$$

$$14. \lim_{x \rightarrow +\infty} \frac{7x}{4x+1+\sqrt{16x^2+x+1}}$$

$$15. \lim_{x \rightarrow +\infty} (3x - \sqrt{x^2 - x + 1})$$

16. $\lim_{x \rightarrow -\infty} (3x - \sqrt{x^2 - x + 1})$
17. $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+x+1} + \sqrt{x^2-x+1}}{x + \sqrt{x^2+1}}$
18. $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+x+1} + \sqrt{x^2-x+1}}{x + \sqrt{x^2+1}}$
19. $\lim_{x \rightarrow +\infty} (2x - 1 - \sqrt{4x^2 - 4x + 1})$
20. $\lim_{x \rightarrow -\infty} (2x - 1 - \sqrt{4x^2 - 4x + 1})$

Réponses

Exercice 1:

1. $\lim_{x \rightarrow -\infty} (x^3 - 2x + 5) = -\infty$
2. $\lim_{x \rightarrow -\infty} (x^4 + 2x) = +\infty$
3. $\lim_{x \rightarrow +\infty} (x^3 - x^4) = -\infty$
4. $\lim_{x \rightarrow -1} (x^3 - 5x + 1) = 5$
5. $\lim_{x \rightarrow +\infty} \left(\frac{-3}{x^2+5}\right) = 0$
6. $\lim_{x \rightarrow 0} \left(\frac{x^2}{x+2}\right) = 0$
7. $\lim_{x \rightarrow 1} \left(\frac{x^2-1}{x^2+3x+5}\right) = 0$
8. $\lim_{x \rightarrow 0} \left(\frac{x^3-1}{x^2}\right) = -\infty$
9. $\lim_{x \rightarrow 2} \left(\frac{x^2+5}{(x-2)^2}\right) = +\infty$
10. $\lim_{x \rightarrow 1} \left(\frac{3-x}{x^2-2x+1}\right) = +\infty$
11. $\lim_{x \rightarrow 1} \left(\frac{3x-2}{x+1}\right) = \frac{1}{2}$
12. $\lim_{x \rightarrow +\infty} \left(\frac{2x^2+5}{x^2-3x+1}\right) = 2$
13. $\lim_{x \rightarrow +\infty} \left(\frac{3x^2+5x}{1-3x^2}\right) = -1$
14. $\lim_{x \rightarrow +\infty} \left(\frac{x+3}{2x^2+5}\right) = 0$

15. $\lim_{x \rightarrow -\infty} \left(\frac{3-x}{x^3+4x+1} \right) = 0$
16. $\lim_{x \rightarrow -\infty} \left(\frac{x^4+3x}{x+3} \right) = -\infty$
17. $\lim_{x \rightarrow +\infty} \left(\frac{x^2+2x+1}{x} \right) = +\infty$
18. $\lim_{x \rightarrow +\infty} \left(\frac{x^3}{x^2+3x+5} \right) = +\infty$
19. $\lim_{x \rightarrow -\infty} \left(\frac{x^2+1}{1-x} \right) = +\infty$
20. $\lim_{x \rightarrow 0} \left(\frac{x^3-3x}{x^2+x} \right) = -3$
21. $\lim_{x \rightarrow 1} \left(\frac{x^2-x}{x^2+2x-3} \right) = \frac{1}{4}$
22. $\lim_{x \rightarrow -2} \left(\frac{x^2+4x+4}{x^2-x-6} \right) = 0$
23. $\lim_{x \rightarrow 2} \left(\frac{3x+5}{(x-2)^2} \right) = +\infty$
24. $\lim_{x \rightarrow -3} \left(\frac{1-3x}{(3+x)^2} \right) = +\infty$
25. $\lim_{x \rightarrow 1} \left(\frac{x-1}{\sqrt{x}-1} \right) = 2$
26. $\lim_{x \rightarrow 4} \left(\frac{2-\sqrt{x}}{4-x} \right) = \frac{1}{4}$

Exercice 2:

1. $\lim_{x \rightarrow 2^-} \frac{3x-7}{x-2} = +\infty$
 $\lim_{x \rightarrow 2^+} \frac{3x-7}{x-2} = -\infty$
2. $\lim_{x \rightarrow -3^-} \frac{5x}{x+3} = +\infty$
 $\lim_{x \rightarrow -3^+} \frac{5x}{x+3} = -\infty$
3. $\lim_{x \rightarrow -2^-} \frac{-3x+2}{x+2} = -\infty$
 $\lim_{x \rightarrow -2^+} \frac{-3x+2}{x+2} = +\infty$
4. $\lim_{x \rightarrow 1^-} \frac{3x}{1-x} = +\infty$
 $\lim_{x \rightarrow 1^+} \frac{3x}{1-x} = -\infty$
5. $\lim_{x \rightarrow 2^-} \frac{2x-5}{2-x} = -\infty$
 $\lim_{x \rightarrow 2^+} \frac{2x-5}{2-x} = +\infty$

6. $\lim_{x \rightarrow 0^-} \frac{x^2+3x+5}{x^2+2x} = -\infty$
 $\lim_{x \rightarrow 0^+} \frac{x^2+3x+5}{x^2+2x} = +\infty$
7. $\lim_{x \rightarrow -2^-} \frac{x^2+3x+5}{x^2+2x} = +\infty$
 $\lim_{x \rightarrow -2^+} \frac{x^2+3x+5}{x^2+2x} = -\infty$
8. $\lim_{x \rightarrow 1^-} \frac{2x}{(x-1)(x+3)} = -\infty$
 $\lim_{x \rightarrow 1^+} \frac{2x}{(x-1)(x+3)} = +\infty$
9. $\lim_{x \rightarrow -3^-} \frac{2x+1}{(x-1)(x+3)} = -\infty$
 $\lim_{x \rightarrow -3^+} \frac{2x+1}{(x-1)(x+3)} = +\infty$
10. $\lim_{x \rightarrow -1^-} \frac{3x+7}{(x+1)(2-x)} = -\infty$
 $\lim_{x \rightarrow -1^+} \frac{3x+7}{(x+1)(2-x)} = +\infty$
11. $\lim_{x \rightarrow 3^-} \frac{x+2}{6+x-x^2} = +\infty$
 $\lim_{x \rightarrow 3^+} \frac{x+2}{6+x-x^2} = -\infty$

Exercice 3:

1. $\lim_{x \rightarrow 1} \frac{x^2-5x+4}{x^2-3x+2} = 3$
2. $\lim_{x \rightarrow 1} \frac{x^3-3x+2}{2x^3-3x^2+1} = 1$
3. $\lim_{x \rightarrow 2} \left[\frac{x+3}{x^2+3x-10} \left(\frac{8}{5(x+2)} - \frac{4}{x} + \frac{17x+6}{5(x^2+1)} \right) \right] = \frac{3}{14}$
4. $\lim_{x \rightarrow 1} \frac{3x-2-\sqrt{4x^2-x-2}}{x^2-3x+2} = \frac{1}{2}$
5. $\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-\sqrt{x^2+x+1}}{x} = 0$
6. $\lim_{x \rightarrow 1} \frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x-1}} = 0$
7. $\lim_{x \rightarrow 2} \frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3} = \frac{9}{8}$
8. $\lim_{x \rightarrow 0} \frac{x^4+1-\sqrt{1-x^4}}{x^2\sqrt{1-x^4}} = 0$
9. $\lim_{x \rightarrow 1} \frac{\sqrt{3+\sqrt{2x-1}}-2}{\sqrt{2+\sqrt{3x+1}}-\sqrt{x+3}} = -4$

10. $\lim_{x \rightarrow 0^+} \frac{x}{\sqrt{1+x^2}-1} = +\infty$
11. $\lim_{x \rightarrow 0^-} \frac{x}{\sqrt{1+x^2}-1} = -\infty$
12. $\lim_{x \rightarrow +\infty} \frac{(3x^2+1)(5x+3)}{(2x^3-1)(x+4)} = 0$
13. $\lim_{x \rightarrow +\infty} \frac{(2x-3)^2(4x+7)^3}{(3x-4)^2(5x^2+1)} = +\infty$
14. $\lim_{x \rightarrow +\infty} \frac{7x}{4x+1+\sqrt{16x^2+x+1}} = \frac{7}{8}$
15. $\lim_{x \rightarrow +\infty} (3x - \sqrt{x^2 - x + 1}) = +\infty$
16. $\lim_{x \rightarrow -\infty} (3x - \sqrt{x^2 - x + 1}) = -\infty$
17. $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{x^2+1}} = 1$
18. $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{x^2+1}} = +\infty$
19. $\lim_{x \rightarrow +\infty} (2x - 1 - \sqrt{4x^2 - 4x + 1}) = 0$
20. $\lim_{x \rightarrow -\infty} (2x - 1 - \sqrt{4x^2 - 4x + 1}) = -\infty$

(Saisie et mise en pages: Diane DOCKENDORF, *II^eC5*, LCD)