

# Dérivées

## Exercice

Calculer les fonctions dérivées des fonctions numériques  $f$  définies, dans  $\mathbb{R}$ .

1.  $f(x) = \frac{x-2}{3-x}$

2.  $f(x) = \frac{2x+3}{4-x}$

3.  $f(x) = \frac{x-x^3}{2-x}$

4.  $f(x) = \frac{x^2+4}{2-x}$

5.  $f(x) = \frac{x^3-x^2}{4-x}$

6.  $f(x) = \frac{x-7}{x^2-3}$

7.  $f(x) = \frac{4-x^2}{x+7}$

8.  $f(x) = \frac{4-x^2}{x-5}$

9.  $f(x) = \frac{8x^2-7}{3x-5}$

10.  $f(x) = (x^2-3)(4x-5)$

11.  $f(x) = (3x^2-7x)(4x^2-5)$

12.  $f(x) = (7x^2-3x)(x^4-2x^3)$

13.  $f(x) = (x^2-3)(2x+4)$

14.  $f(x) = (x^3-1)(4x^2+x-2)$

15.  $f(x) = (8x^2+3x-1)(2x^2-4)$

16.  $f(x) = (4x^2-7)(3-5x)$

17.  $f(x) = (x^2+3)(3-4x^2)$

$$18. f(x) = \frac{(x^2 - 3x)(4x + 2)}{x - 5}$$

$$19. f(x) = \frac{(4x - 5)(7x - 2)}{3x^2 - 1}$$

$$20. f(x) = \frac{(x - 5)(3 - 2x)}{4x + 2}$$

$$21. f(x) = \frac{(x^2 - 3x)(4x - 5)}{2x^3 - 5x + 2}$$

$$22. f(x) = \frac{(x - 7)(x^3 - 5)}{4 - x^2}$$

$$23. f(x) = \frac{(7 - x)(3 - x^2)}{x^3 - 1}$$

$$24. f(x) = \frac{(x - 4)(3x - 7)}{x^2 - 4x + 2}$$

$$25. f(x) = (x - 7)(3x + 2)(4x^2 - 3)$$

$$26. f(x) = 5\sqrt{3x^4 + 5x + 2}$$

$$27. f(x) = 8\sqrt{2x^3 - 5}$$

$$28. f(x) = 7\sqrt{4x^2 + 3}$$

$$29. f(x) = (x^2 + 3)^3(2x + 4)$$

$$30. f(x) = (4x^3 - 5)^2(2x^3 + 1)$$

$$31. f(x) = (3x^2 + 5)(4x + 7)$$

$$32. f(x) = \frac{(3^2 + 5)^2}{x^3 + 4}$$

$$33. f(x) = \frac{7x^2 + 3}{(2x^3 + 4x)^2}$$

$$34. f(x) = \frac{4x^2 + 5}{(3x + 2)^3}$$

$$35. f(x) = \frac{(2x + 4)3x^2 + 5^2}{3x - 2}$$

$$36. f(x) = \frac{\sqrt{3x^4 - 5}}{(2x^2 + 3)^2}$$

$$37. f(x) = (5x^2 - 6x + 9)\sqrt{(x + 1)^2}$$

38.  $f(x) = \sqrt{\frac{1-x}{1+x}}$

# Solutions

$$1. f'(x) = \frac{1}{3-x} + \frac{x-2}{(3-x)^2} = \frac{1}{(-3+x)^2}$$

$$2. f'(x) = \frac{2}{4-x} + \frac{2x+3}{(4-x)^2} = \frac{11}{(x-4)^2}$$

$$3. f'(x) = \frac{1-3x^2}{2-x} + \frac{x-x^3}{(2-x)^2} = \frac{2(x^3-3x^2+1)}{(x-2)^2}$$

$$4. f'(x) = 2 \cdot \frac{x}{2-x} + \frac{x^2+4}{(2-x)^2} = -\frac{x^2-4x-4}{(x-2)^2}$$

$$5. f'(x) = \frac{3x^2-2x}{4-x} + \frac{x^3-x^2}{(4-x)^2} = -\frac{x(2x^2-13x+8)}{(x-4)^2}$$

$$6. f'(x) = \frac{1}{x^2-3} - 2 \cdot \frac{x-7}{(x^2-3)^2} \cdot x = -\frac{x^2+3-14x}{(x^2-3)^2}$$

$$7. f'(x) = -2 \cdot \frac{x}{x+7} - \frac{4-x^2}{(x+7)^2} = -\frac{x^2+14x+4}{(x+7)^2}$$

$$8. f'(x) = -2 \cdot \frac{x}{x-5} - \frac{4-x^2}{(x-5)^2} = -\frac{x^2-10x+4}{(x-5)^2}$$

$$9. f'(x) = 16 \cdot \frac{x}{3x-5} - 3 \cdot \frac{8x^2-7}{(3x-5)^2} = \frac{24x^2-80x+21}{(3x-5)^2}$$

$$10. f'(x) = 2x(4x-5) + 4x^2 - 12 = 12x^2 - 10x - 12$$

$$11. f'(x) = (6x-7)(4x^2-5) + 8(3x^2-7x)x = 48x^3 - 30x - 84x^2 + 35$$

$$12. f'(x) = (14x-3)(x^4-2x^3) + (7x^2-3x)(4x^3-6x^2) \\ = 42x^5 - 85x^4 + 24x^3$$

$$13. f'(x) = 2x(2x+4) + 2x^2 - 6 = 6x^2 + 8x - 6$$

$$14. f'(x) = 3x^2(4x^2+x-2) + (x^3-1)(8x+1) = 20x^4 + 4x^3 - 6x^2 - 8x - 1$$

$$15. f'(x) = (16x+3)(2x^2-4) + 4(8x^2+3x-1)x = 64x^3 - 68x + 18x^2 - 12$$

$$16. f'(x) = 8x(3-5x) - 20x^2 + 35 = 24x - 60x^2 + 35$$

$$17. f'(x) = 2x(3-4x^2) - 8(x^2+3)x = -18x - 16x^3$$

18.  $f'(x) = (2x - 3) \frac{4x + 2}{x - 5} + 4 \frac{x^2 - 3x}{x - 5} - (x^2 - 3x) \frac{4x + 2}{(x - 5)^2}$   
 $= \frac{2(4x^3 - 35x^2 + 50x + 15)}{(x - 5)^2}$
19.  $f'(x) = 4 \frac{7x - 2}{3x^2 - 1} + 7 \frac{4x - 5}{3x^2 - 1} - 6(4x - 5) \frac{7x - 2}{(3x^2 - 1)^2}$   
 $= \frac{-116x + 129x^2 + 43}{(3x^2 - 1)^2}$
20.  $f'(x) = \frac{3 - 2x}{4x + 2} - 2 \cdot \frac{x - 5}{4x + 2} - 4(x - 5) \cdot \frac{3 - 2x}{(4x + 2)^2} = -\frac{4x^2 + 4x - 43}{2(2x + 1)^2}$
21.  $f'(x) = \frac{(2x - 3)(4x - 5)}{2x^3 - 5x + 2} + \frac{4(x^2 - 3x)}{2x^3 - 5x + 2} - \frac{(x^2 - 3x)(4x - 5)(6x^2 - 5)}{(2x^3 - 5x + 2)^2}$   
 $= \frac{-100x^3 + 109x^2 + 34x^4 - 68x + 30}{(2x^3 - 5x + 2)^2}$
22.  $f'(x) = \frac{x^3 - 5}{4 - x^2} + 3(x - 7) \frac{x^2}{4 - x^2} + 2(x - 7) \cdot \frac{x^3 - 5}{(4 - x^2)^2} \cdot x$   
 $= -\frac{-16x^3 + 2x^5 + 20 + 89x^2 - 7x^4 - 70x}{(-4 + x^2)^2}$
23.  $f'(x) = -\frac{3 - x^2}{x^3 - 1} - 2(7 - x) \cdot \frac{x}{x^3 - 1} - 3(7 - x) \cdot \frac{3 - x^2}{(x^3 - 1)^2} \cdot x^2$   
 $= \frac{6x^3 + 3 - 66x^2 + 7x^4 + 14x}{(x^3 - 1)^2}$
24.  $f'(x) = \frac{3x - 7}{x^2 - 4x + 2} + 3 \cdot \frac{x - 4}{x^2 - 4x + 2} - (x - 4) \cdot \frac{3x - 7}{(x^2 - 4x + 2)^2} \cdot (2x - 4)$   
 $= \frac{7x^2 - 44x + 74}{(x^2 - 4x + 2)^2}$
25.  $f'(x) = (3x + 2)(4x^2 - 3) + 3(x - 7)(4x^2 - 3) + 8(x - 7)(3x + 2)x$   
 $= 48x^3 - 130x - 228x^2 + 57$
26.  $f'(x) = \frac{5(12x^3 + 5)}{2\sqrt{(3x^4 + 5x + 2)}}$
27.  $f'(x) = \frac{24x^2}{\sqrt{(2x^3 - 5)}}$
28.  $f'(x) = \frac{28x}{\sqrt{(4x^2 + 3)}}$

29.  $f'(x) = 6(x^2 + 3)^2(2x + 4)x + 2(x^2 + 3)^3$   
 $= 14x^6 + 24x^5 + 90x^4 + 144x^3 + 162x^2 + 216x + 54$
30.  $f'(x) = 24(4x^3 - 5)(2x^3 + 1)x^2 + 6(4x^3 - 5)^2x^2 = 288x^8 - 384x^5 + 30x^2$
31.  $f'(x) = 6x(4x + 7) + 12x^2 + 20 = 36x^2 + 42x + 20$
32.  $f'(x) = -\frac{588x^2}{(x^3 + 4)^2}$
33.  $f'(x) = 14 \cdot \frac{x}{(2x^3 + 4x)^2} - 2 \cdot \frac{7x^2 + 3}{(2x^3 + 4x)^3} (6x^2 + 4) = -\frac{14x^4 + 9x^2 + 6}{2[x^3(x^2 + 2)^3]}$
34.  $f'(x) = 8 \cdot \frac{x}{(3x + 2)^3} - 9 \cdot \frac{4x^2 + 5}{(3x + 2)^4} = -\frac{12x^2 - 16x + 45}{(3x + 2)^4}$
35.  $f'(x) = \frac{6x^2 + 6(2x + 4)x}{3x - 2} - 3 \cdot \frac{3(2x + 4)x^2 + 5}{(3x - 2)^2} = \frac{3(12x^3 - 16x - 5)}{(3x - 2)^2}$
36.  $f'(x) = \frac{6}{\sqrt{(3x^4 - 5)}(2x^2 + 3)^2} \cdot x^3 - 8 \cdot \frac{\sqrt{(3x^4 - 5)}}{(2x^2 + 3)^3} \cdot x$   
 $= -\frac{2x(6x^4 - 9x^2 - 20)}{\sqrt{(3x^4 - 5)}(2x^2 + 3)^3}$
37.  $f'(x) = (10x - 6)\sqrt{(x + 1)^2} + \frac{1}{2} \cdot \frac{5x^2 - 6x + 9}{\sqrt{(x + 1)^2}} \cdot (2x + 2) = 15x^2 - 2x + 3$
38.  $f'(x) = \frac{1}{2\sqrt{\left(\frac{1-x}{x+1}\right)}} \left( -\frac{1}{x+1} - \frac{1-x}{(x+1)^2} \right) = -\frac{1}{(2x + x^2 + 1)\sqrt{\frac{1-x}{x+1}}}$

(Saisie et mise en pages: Christophe THEIS; Iere B, LCD)