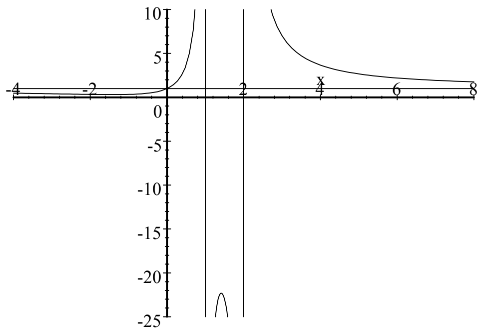
Etude de fonctions:
solutions succintes

1. $f(x) = \dfrac{x^2+x+2}{x^2-3x+2}$

   domf=$\mathbb{R}\setminus\{1,2\}$=domf'=domf"

   $A.H.: y=1; A.V.: x=1; A.V.: x=2$

   $f'(x) = \dfrac{-4(x^2-2)}{(x^2-3x+2)^2}; f''(x) = \dfrac{8(x^3-6x+6)}{(x^2-3x+2)^3}$

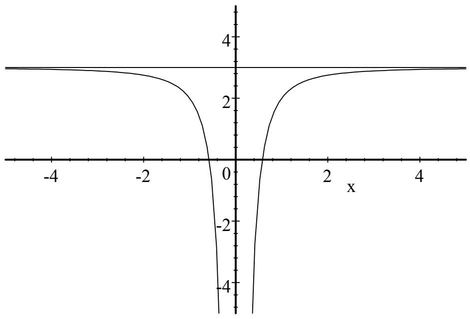


2. $f(x) = 3 - \dfrac{1}{x^2}$

   domf=$\mathbb{R}^*$=domf'=domf"

   $A.H.: y=3; A.V.: x=0$

   $f'(x) = \dfrac{2}{x^3}; f''(x) = -\dfrac{6}{x^4}$

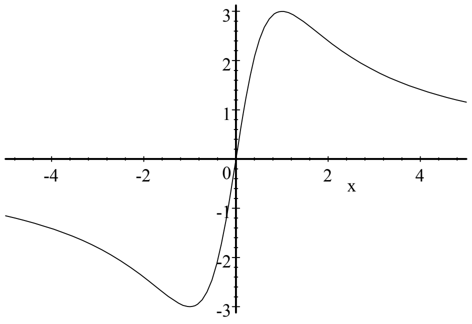


3. $f(x) = \dfrac{6x}{x^2+1}$

   domf=$\mathbb{R}$=domf'=domf"

   $A.H. : y = 0$

   $f'(x) = \dfrac{-6(x^2-1)}{(x^2+1)^2}; f''(x) = \dfrac{12x(x^2-3)}{(x^2+1)^3}$

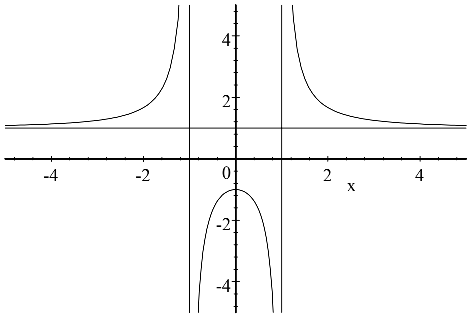


4. $f(x) = \dfrac{x^2+1}{x^2-1}$

   domf=$\mathbb{R}\setminus\{1,-1\}$=domf'=domf"

   $A.H. : y = 1; A.V. : x = -1; A.V. : x = 1$

   $f'(x) = \dfrac{-4x}{(x^2-1)^2}; f''(x) = \dfrac{4(3x^2+1)}{(x^2-1)^3}$

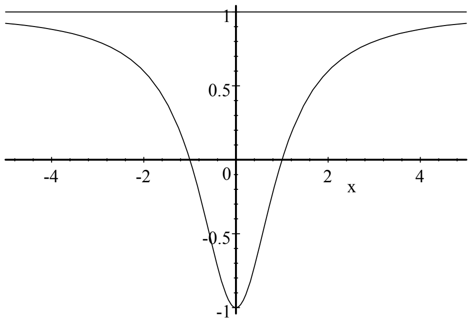


5. $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

   domf=$\mathbb{R}$=domf'=domf"

   $A.H. : y = 1$

   $f'(x) = \dfrac{4x}{(x^2 + 1)^2}; f''(x) = \dfrac{-4(3x^2 - 1)}{(x^2 + 1)^3}$

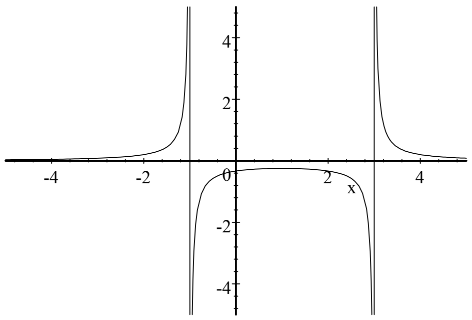


6. $f(x) = \dfrac{1}{x^2 - 2x - 3}$

   domf=$\mathbb{R}\setminus\{-1, 3\}$=domf'=domf"

   $A.H. : y = 0; A.V. : x = -1; A.V. : x = 3$

   $f'(x) = \dfrac{-2(x - 1)}{(x^2 - 2x - 3)^2}; f''(x) = \dfrac{2(3x^2 - 6x + 7)}{(x^2 - 2x - 3)^3}$



7. $f(x) = \dfrac{x^3 + 2x - 3}{x^2 + 3x - 4}$

   domf=$\mathbb{R}\setminus\{1, -4\}$=domf'=domf"

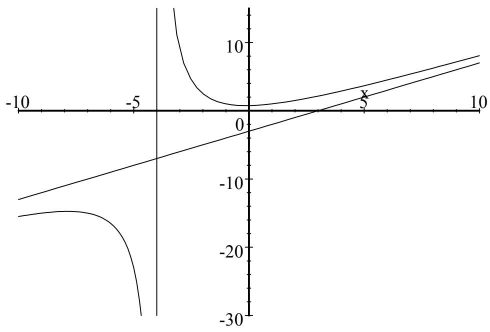
   Trouver a,b,c et d réels tels que $\forall x \in$domf: $f(x) = ax + b + \dfrac{cx + d}{x^2 + 3x - 4}$

On trouve:

$f(x) = x - 3 + \dfrac{15x - 15}{x^2 + 3x - 4}$

$A.V. : x = -1; A.V. : x = 1; A.O. : y = x - 3$

$f'(x) = \dfrac{x^2 + 8x + 1}{(x+4)^2}; f''(x) = \dfrac{30}{(x+4)^3}$



8. $f(x) = \dfrac{x^3 - x + 1}{x^2}$

domf=$\mathbb{R}^*$=domf'=domf"

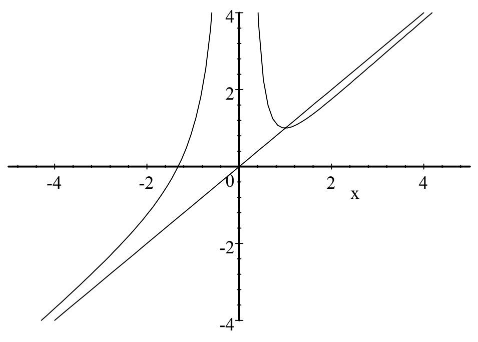
Trouver a,b,c et d réels tels que $\forall x \in$domf: $f(x) = ax + b + \dfrac{cx + d}{x^2}$

On trouve:

$f(x) = x + \dfrac{1 - x}{x^2}$

$A.V. : x = 0; A.O. : y = x$

$f'(x) = \dfrac{x^3 + x - 2}{x^3}; f''(x) = -\dfrac{2(x-3)}{x^4}$

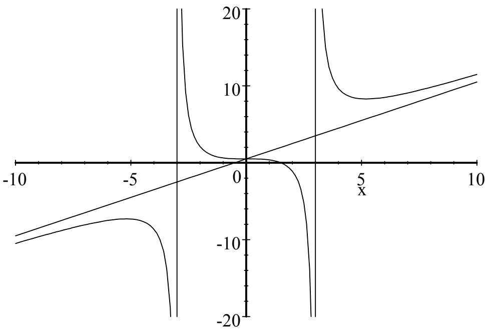


9. $f(x) = x + \frac{1}{2} + \frac{9x}{x^2 - 9}$

domf=$\mathbb{R}\setminus\{-3, 3\}$=domf'=domf"

$A.V.: x = -3; A.V.: x = 3; A.O.: y = x + \frac{1}{2}$

$f'(x) = \frac{x^2(x^2-27)}{(x^2-9)^2}; f''(x) = \frac{18x(x^2+27)}{(x^2-9)^3}$



10. $f(x) = \frac{2}{x(x+1)(x+2)}$

domf=$\mathbb{R}^*\setminus\{-1, -2\}$=domf'=domf"

$A.H.: y = 0; A.V.: x = -2; A.V.: x = -1; A.V.: x = 0$

$f'(x) = \frac{-2(3x^2+6x+2)}{x^2(x+1)^2(x+2)^2}; f''(x) = \frac{4(6x^4+24x^3+33x^2+18x+4)}{x^3(x+1)^3(x+2)^3}$